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EXPERIMENTAL INVESTIGATION OF LAMINAR FREE-CONVECTION FLOW IN AIR ABOVE HORIZONTAL WIRE WITH CONSTANT HEAT FLUX

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Abstract—Experimentally determined velocity and temperature distributions are presented for free-convection flow above a horizontal wire in air, with constant heat flux. The method used in the velocity measurements is a technique of visualization of streamlines by means of small dust particles introduced into the air.

The temperature distribution is determined from an interferogram.

NOMENCLATURE

- x, vertical co-ordinate;
- y, horizontal co-ordinate;
- u, velocity component along x;
- v, velocity component along y;
- c_p, specific heat of the fluid at constant pressure;
- ρ, density;
- T, temperature;
- T_∞, ambient temperature;
- β, coefficient of cubic thermal expansion;
- ν, kinematic viscosity;
- g, gravitational acceleration;
- a, thermal diffusivity;
- Θ_L, number with dimension of temperature;
- Q_L, heat input to the wire;
- Q, heat transported by trace;
- G_L, modified Grashof number;
- ξ, independent variable of the "similar" solution;
- f, dimensionless stream function;
- h, dimensionless temperature.

1. INTRODUCTION

THE PAPER presents the results of an experimental investigation of the velocity and temperature fields above and around a thin horizontally stretched wire heated electrically.

Mass and heat balances calculated from the experimental data confirm the accuracy of the quantities used in these balances. The heated wire in the experiment corresponds to the theoretical problem of the laminar wake above a line heat source with constant heat flux per unit length. This theoretical problem was investigated by Schuh [3] and more recently by Fujii [1], with whose results the experimental data are compared.

Certain differences between the experimental and theoretical results may result from simplifications in the partial differential equations used by Fujii. These simplifications are similar to those of boundary-layer problems, as will be shown below.

2. DESCRIPTION OF APPARATUS AND METHODS OF MEASUREMENTS

A wire 0·075 mm O.D. and 250 mm long was stretched horizontally and heated by a direct electric current passing through it. The heat input was adjusted by a resistor in series and was measured by means of a wattmeter of high accuracy.

The velocity distribution was determined by a streamline visualization technique described in detail by the authors [2]. In this technique small dust particles are introduced into the air around the wire. A thin light beam illuminating the region of interest is directed in a plane perpendicular to the wire. The illuminated plane is placed sufficiently far from the ends of the wire so that the investigated velocity field is two-dimensional. The third velocity component parallel to the wire, which exists because of the limited length of the wire, is likely to be of significance only near to planes perpendicular to the wire and passing through its ends. A dust particle trajectory in the velocity field is similar to a streamline. On passing through the lighted area, a dust particle shines with reflected light. The trajectories of dust particles can thus be recorded by photographs with a suitable time of exposure. By using stroboscopes (two, to cover the entire velocity range) the light beam can be rendered intermittent, and consequently the trajectories are dotted. Since the intercept time interval is known, velocity components can be determined from the distance between two positions of a dust particle on the photograph.

The temperature distribution was determined by means of the Mach–Zehnder interferometer.

3. EXPERIMENTAL RESULTS

The conditions under which the photographs were taken are as follows:

Ambient temperature $T_\infty = 19{\cdot}8°C$, heat supply to the wire $Q_L = 9{\cdot}75$ W/m. The velocity distribution was obtained from the photograph shown in Fig. 1. For arbitrarily chosen levels above the wire, at 1, 2, 4 and 8 cm, the velocity components are given in Tables 3–6 (see Appendix) and in graphs plotted from these Tables in Fig. 3.

The temperature distribution in the wake was obtained from the photograph shown in Fig. 2. For arbitrarily chosen values of the x co-ordinate, 1, 4 and 8 cm, see Fig. 4.

The relation between the maxima of the velocity components and the distance above the wire is plotted in Figs. 5 and 6.

4. MASS BALANCE IN THE WAKE

The mass balance in the wake per unit length, neglecting the change of density with temperature, may be written in the form:

$$\int_{y1}^{y2} u(x_j, y)\,dy = \int_{y1}^{y2} u(x_i, y)\,dy + \int_{xi}^{xj} v(x, y_1)\,dx - \int_{xi}^{xj} v(x, y_2)\,dx \quad (1)$$

where subscripts i and j indicate two successive values of the x co-ordinate and y_1 and y_2 two arbitrarily chosen values of y. Each of the above integrals, which may be denoted respectively as in equation (1) by

$$m_j,\ m_i,\ m_{ij}^1,\ m_{ij}^2$$

can be determined graphically provided the velocity field is known.

Denoting the right-hand side of equation (1) by m_j^*,

$$m_j^* = m_i + m_{ij}^1 + m_{ij}^2.$$

It is possible to compare the value m_j^* and m_j, which through experimental error are not exactly equal.

Numerical results for some arbitrary control surfaces are presented in Table 1.

5. HEAT BALANCE IN THE WAKE

The heat balance, including the heat transported by the wake, is

$$Q = \int_{-\infty}^{+\infty} c_p\, \rho(T - T_\infty)\, u\, dy, \quad (2)$$

Table 1. Mass balance in the wake. The values of mass flowing through control planes; x = const; y = const.

x (cm)	m_{ij}^1 $y_1 = -16$ mm (cm³/s)	m_{ij}^2 $y_2 = +16$ mm (cm³/s)	m_j (cm³/s)	m_j^* (cm³/s)	$\frac{m_j - m_j^*}{m_j} \cdot 1000$ (%)
1			7·66	—	—
	1·15	1·15			
2			10·29	9·96	+3·2
	2·10	2·10			
4			15·23	14·49	+4·85
	2·96	2·80			
8			22·00	20·99	+4·60

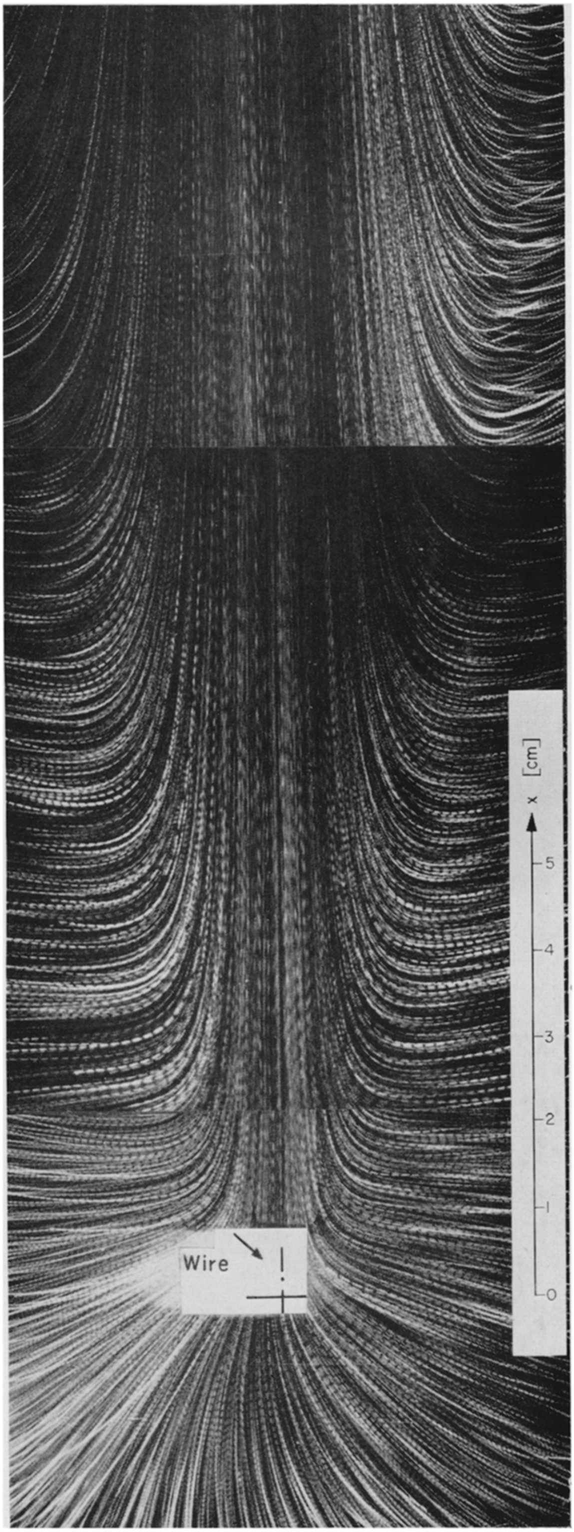

FIG 1(a). Dust trajectories (streamlines) around and above a horizontal wire.

[*facing p.* 82]

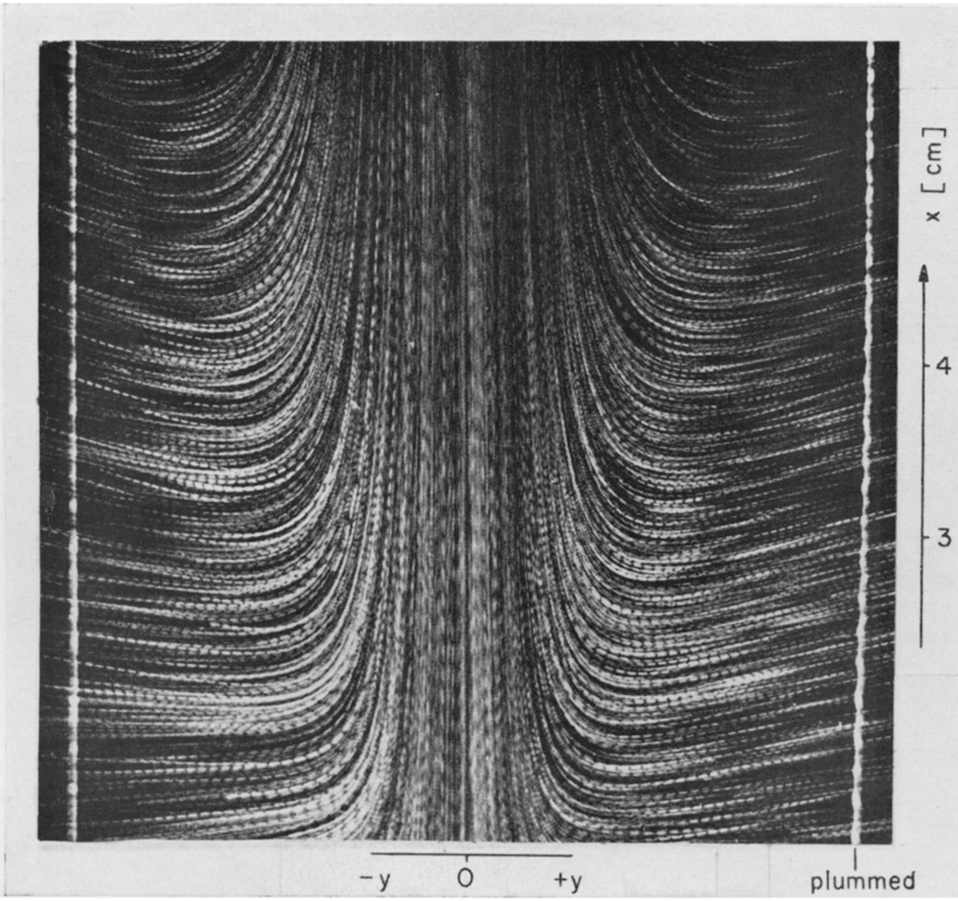

FIG. 1(b) Dust trajectories (streamlines) above a horizontal wire. Stroboscope frequencies $100 s^{-1}$ and $18{\cdot}2 s^{-1}$.

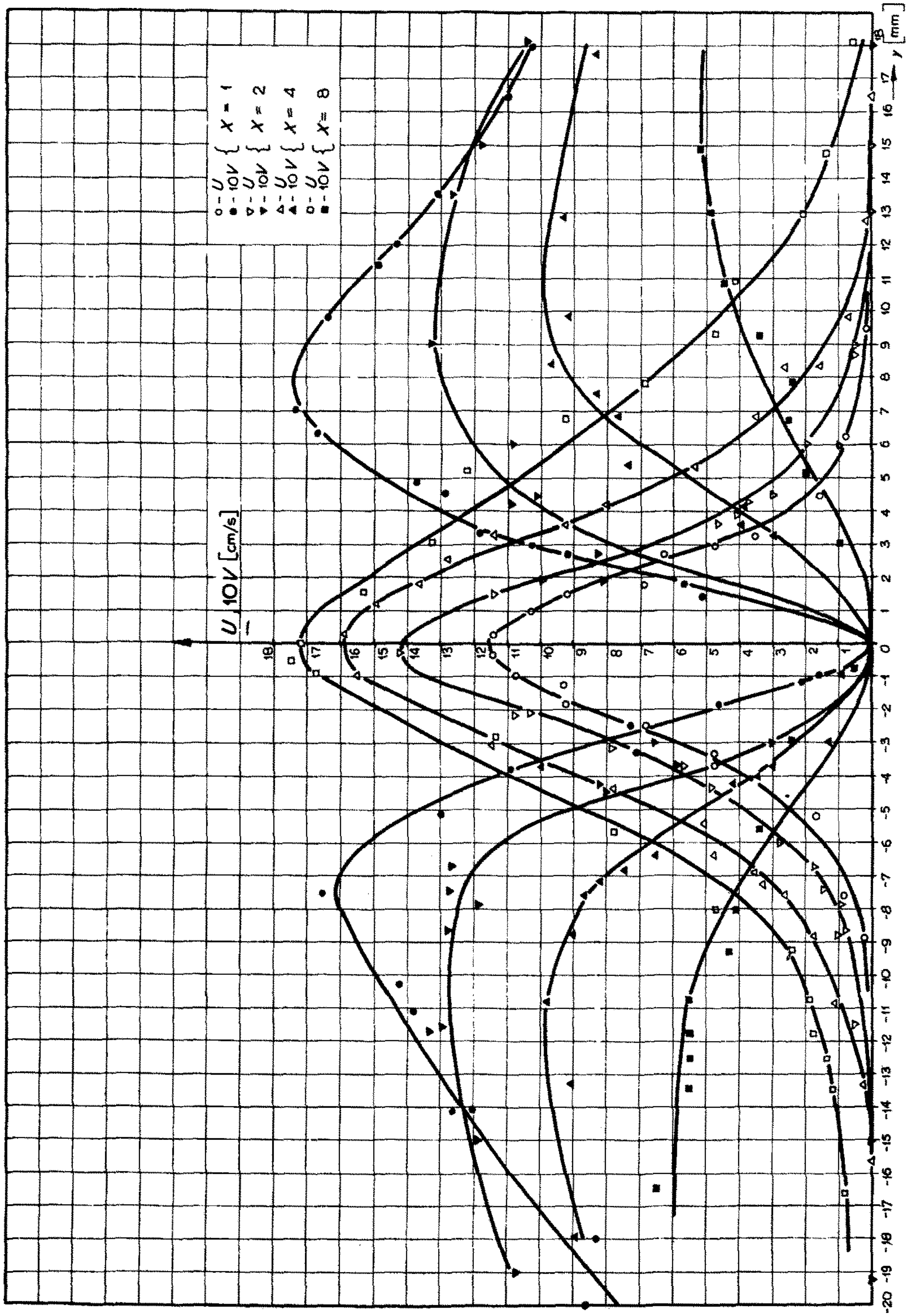

FIG. 3. Velocity distribution in the wake.

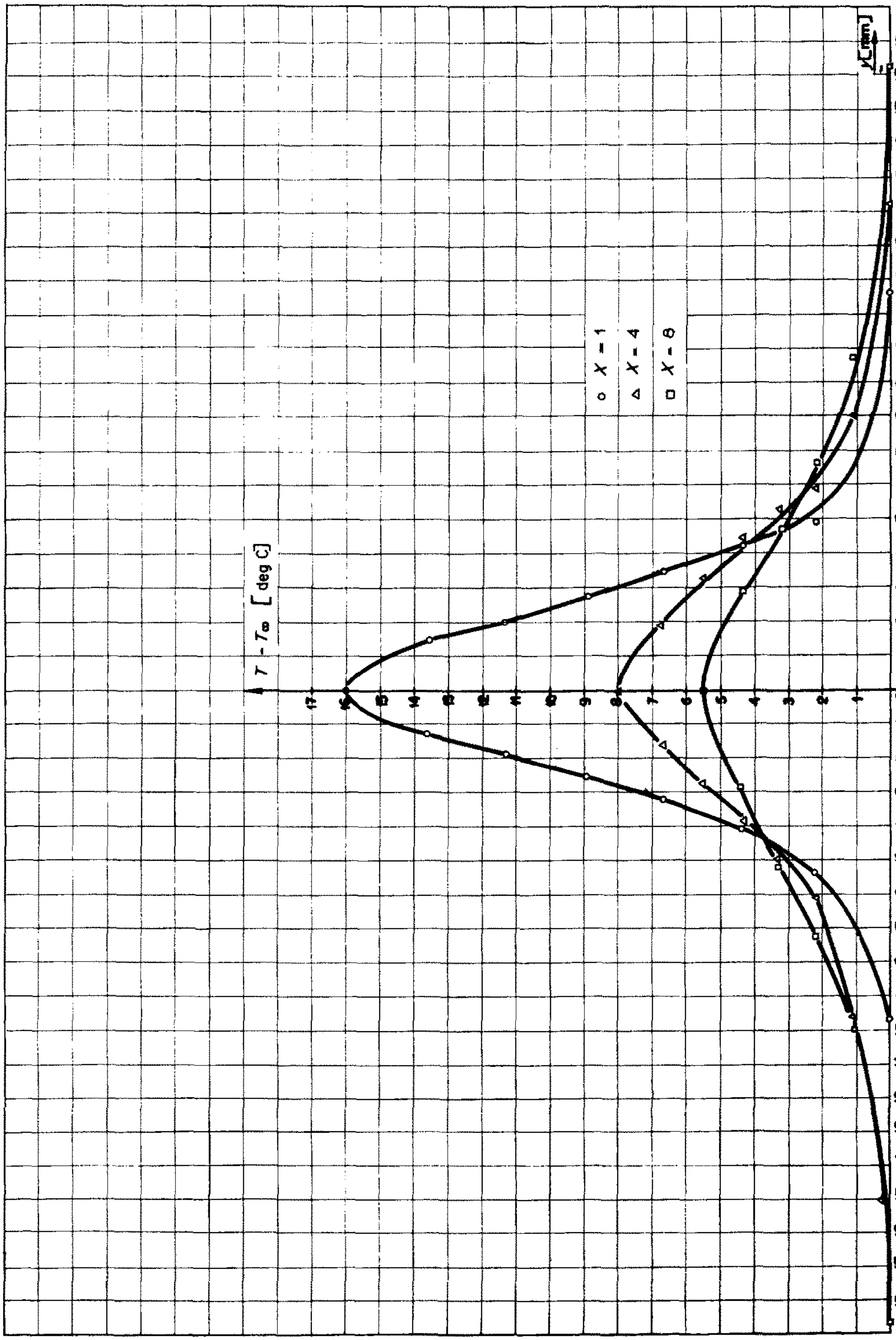

FIG. 4. Temperature distribution in the wake.

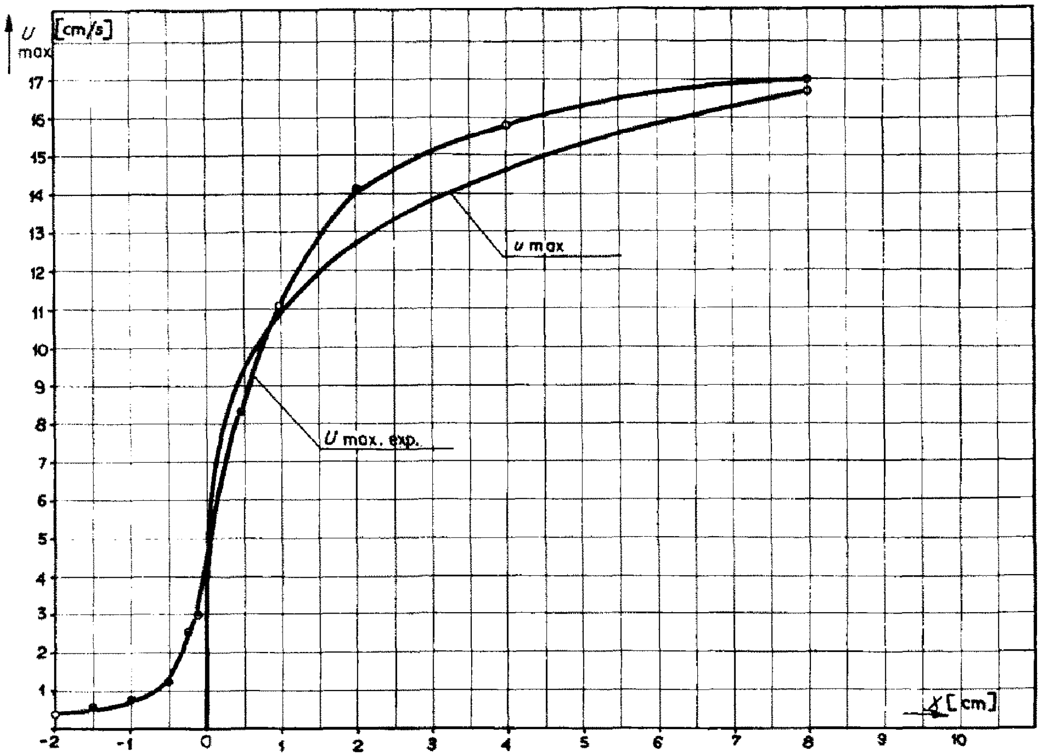

FIG. 5. Distribution of maximum vertical velocity component *u* against the distance from the wire.

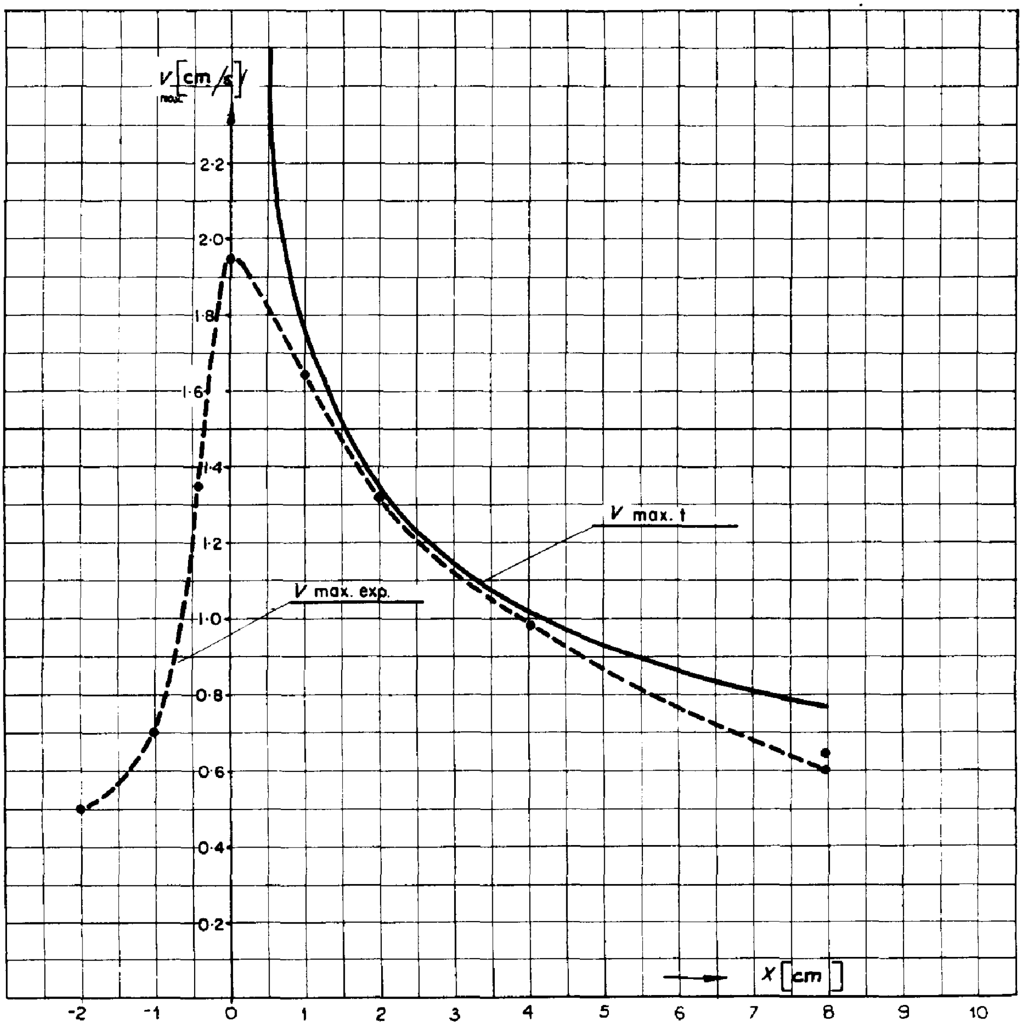

FIG. 6. Distribution of maximum horizontal velocity component u against the distance from the wire.

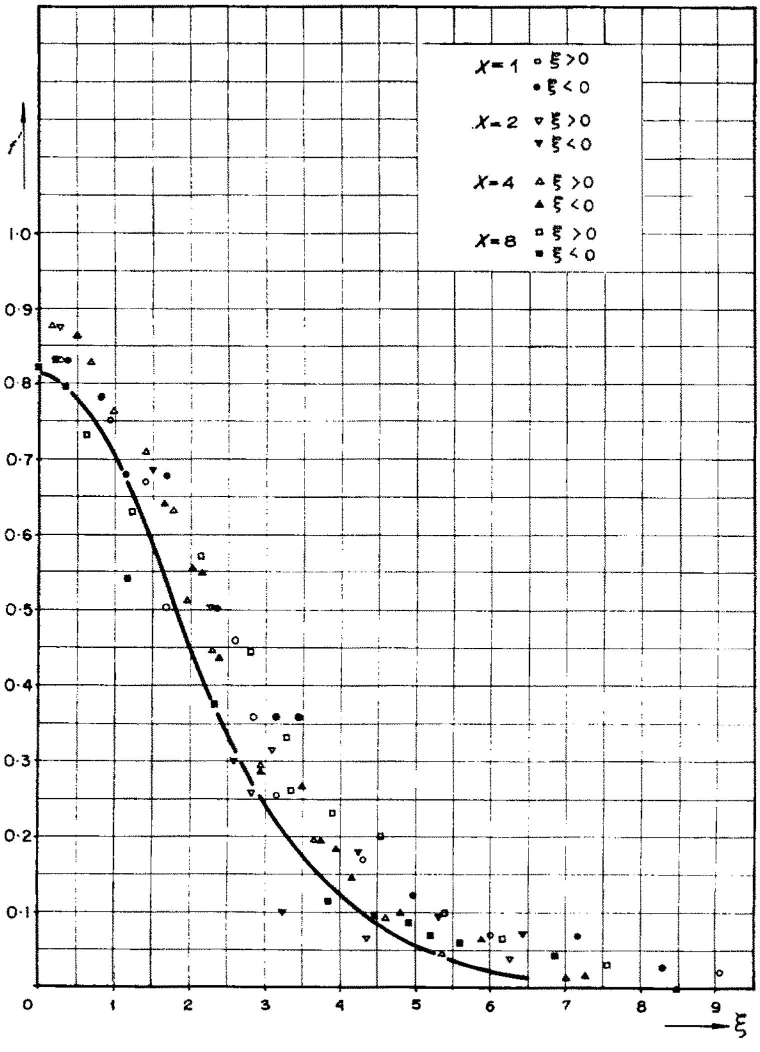

FIG. 7. Values of function f' calculated at points where the velocity and its theoretical distribution was determined from equation 4.

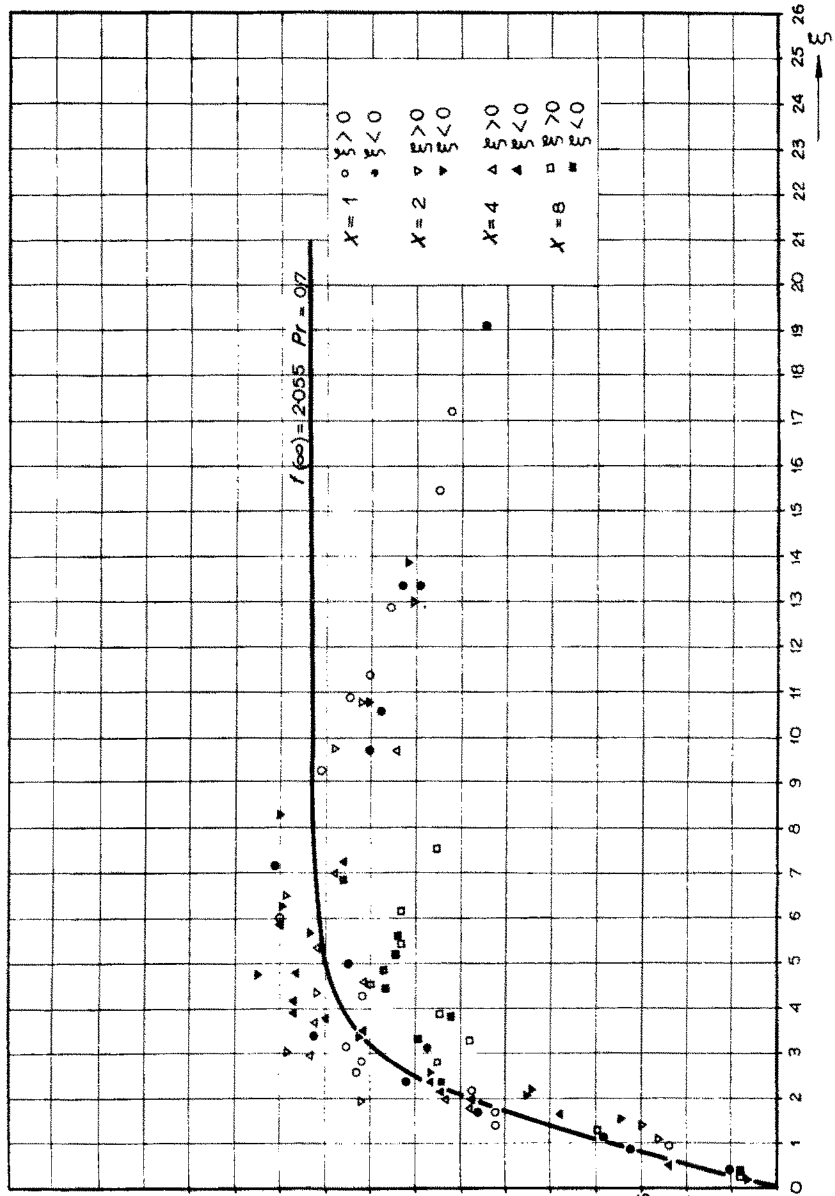

Fig. 8. Values of function f compared with the theoretical ones.

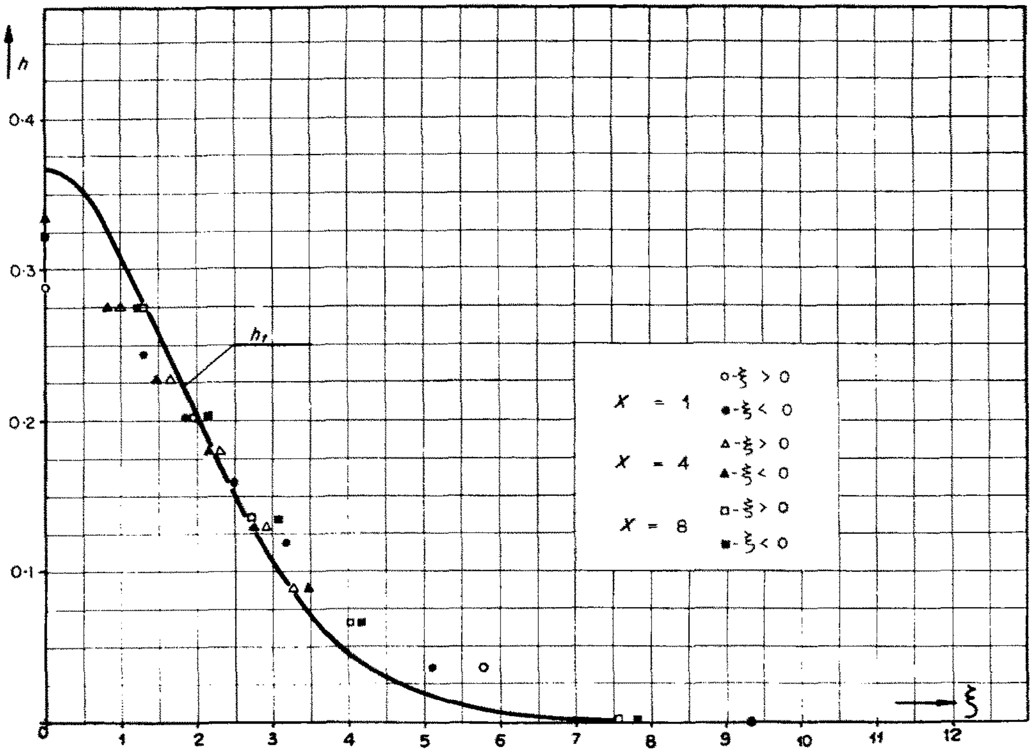

FIG. 9. Values of function h compared with the theoretical ones.

and when the variation of c_p and ρ with temperature is neglected it may be written in a simplified form as

$$Q = c_p\, \rho \int_{-\infty}^{+\infty} (T - T_\infty) u \, dy. \qquad (3)$$

The values of the latter integral have been determined graphically for some arbitrary levels above the wire and are presented in Table 2. Each value should be equal to the heat input to the wire that is

$$Q = Q_L.$$

Experimental inaccuracy produces a small deviation, as shown in Table 2.

Table 2. Heat balance in the wake. The values of heat capacity flowing through planes; $x = const.$

x	Q	$\frac{Q - Q_L}{Q_L} \cdot 100$
(cm)	(w/m)	(%)
1	9·66	−0·9
4	9·77	0·0
8	9·65	−1·0

6. COMPARISON OF EXPERIMENTAL RESULTS WITH THE THEORETICAL SOLUTION

The experimental results will be compared with Fujii's theoretical solution of the following set of partial differential equations:

$$\left.\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2}, \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \\ u\frac{\partial(T - T_\infty)}{\partial x} + v\frac{\partial(T - T_\infty)}{\partial y} &= a\frac{\partial^2(T - T_\infty)}{\partial y^2}, \end{aligned}\right\} \qquad (4)$$

with following boundary conditions:

$$y = 0 : v = 0,\ \frac{\partial u}{\partial y} = 0,\ \frac{\partial(T - T_\infty)}{\partial y} = 0;$$
$$y = \infty : u = 0,\ T = T_\infty. \qquad (5)$$

These may be transformed by the following substitutions:

$$\left.\begin{aligned} \Theta_L &= \frac{Q_L}{c_p \rho \nu}\,[^\circ\mathrm{C}], \quad G_L = \frac{g\,\beta x^3 \Theta_L}{\nu^2}, \\ \xi &= G_L^{1/5}\frac{y}{x}, \quad u\frac{\nu}{x} = G_L^{2/5} f' \\ v &= -\frac{\nu}{x} G_L^{1/5}\left(\tfrac{3}{5} f - \tfrac{2}{5}\xi f'\right), \end{aligned}\right\} \qquad (6)$$

$$T - T_\infty = \Theta_L\, G_L^{-1/5}\, h,$$

to the ordinary differential equations

$$f''' + \tfrac{3}{5} f f'' - \tfrac{1}{5} f'^2 + h = 0,$$
$$h'' + \tfrac{3}{5} Pr(fh)' = 0, \qquad (7)$$

with boundary conditions

$$\xi = 0 : f = 0,\ f'' = 0,\ h' = 0;$$
$$\xi = \infty : f' = 0,\ h = 0 \qquad (8)$$

Provided the heat generated in the wire is known, equations (6) allow us to calculate, for arbitrary values of x and y where both components of velocity and temperature have been determined, the values of the universal functions ξ, f, f' and h at this point. Values of the universal functions calculated in this way, with the free-stream temperature as reference temperature, are incorporated in Tables 3–6 and plotted in Figs. 7–9.

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APPENDIX

Velocity components and universal functions for constant distances above the wire level:

Table 3: $x = 1$ cm; Table 5: $x = 4$ cm;
Table 4: $x = 2$ cm; Table 6: $x = 8$ cm.

Table 3

$x = 1$ cm: $\xi = 0{\cdot}954\, y$; $f' = 0{\cdot}0732\, u$; $f = 1{\cdot}158\, v + \frac{2}{3}\xi f'$

y (mm)	u (cm/s)	v (cm/s)	ξ	f'	f
−20	approx. 0·3	0·87	19·1	0·0218	1·287
−14	„ 0·3	1·20	13·35	0·0218	1·585
−14	„ 0·3	1·26	13·35	0·0218	1·656
−11·1	„ 0·3	1·38	10·58	0·0218	1·754
−10·2	„ 0·3	1·43	9·72	0·0218	1·799
−8·7	0·40	—	8·29	0·0292	—
−7·5	0·90	1·65	7·15	0·0656	2·226
−5·2	1·70	1·30	4·96	0·124	1·917
−3·6	4·80	1·09	3·43	0·357	2·055
−3·3	4·80	0·71	3·14	0·357	1·540
−2·5	6·90	0·73	2·38	0·503	1·643
−1·8	9·30	0·47	1·718	0·678	1·319
−1·2	9·30	0·22	1·144	0·678	0·771
−0·9	10·80	0·17	0·858	0·787	0·646
−0·4	11·4	~0	0·381	0·828	0·210
+0·3	11·4	~0	0·286	0·828	0·158
+1·0	10·3	~0	0·954	0·751	0·476
+1·5	9·2	0·52	1·430	0·671	1·242
+1·8	6·9	0·57	1·718	0·503	1·236
+2·7	6·3	0·92	2·58	0·459	1·856
+3·0	4·8	1·03	2·86	0·357	1·830
+3·3	3·5	1·18	3·14	0·255	1·902
+4·5	1·6	1·29	4·29	0·1167	1·829
+4·7	—	1·37	4·48	—	—
+6·3	0·8	1·66	6·00	0·0683	2·198
+7·0	approx. 0·3	1·73	6·68	0·218	—
+9·5	„ 0·3	—	9·06	0·0218	—
+9·7	„ 0·3	1·63	10·9	0·0218	1·885
+11·4	„ 0·3	1·49	10·9	0·0218	1·885
+12·0	„ 0·3	1·43	11·4	0·0218	1·804
+13·5	„ 0·3	1·31	12·9	0·0218	1·706
+16·4	„ 0·3	1·09	15·6	0·0218	1·490
+18·0	„ 0·3	1·03	17·2	0·0218	1·444

Table 4

$x = 2$ cm: $\xi = 0{\cdot}724\, y$; $f' = 0{\cdot}0638\, u$; $f = 1{\cdot}54\, v + \frac{2}{3}\xi f'$

y (mm)	u (cm/s)	v (cm/s)	ξ	f'	f
−19·2	~0	1·06	13·9	~0	1·627
−15·0	~0	1·18	10·8	~0	1·810
−11·5	0·576	1·30	8·30	0·0366	2·200
−8·65	0·840	1·28	6·25	0·0535	2·190
−7·9	1·000	1·19	5·70	0·0636	2·07
−6·7	1·730	1·27	4·84	0·1100	2·30
−4·5	4·42	0·80	3·25	0·2820	1·838

Table 4—continued

y (mm)	u (cm/s)	v (cm/s)	ξ	f'	f
−3·6	5·77	0·58	2·60	0·3670	1·525
−3·0	6·60	0·30	2·17	0·420	1·066
−2·16	10·30	~0	1·56	0·655	0·680
−0·3	14·3	~0	0·22	0·910	0·133
+1·5	11·4	~0	1·08	0·730	0·524
+1·95	10·0	~0	1·41	0·635	0·596
+2·7	6·9	0·83	1·95	0·439	1·840
+4·2	3·9	1·09	3·03	0·248	2·175
+6·0	1·99	1·08	4·33	0·127	0·026
+9·0	0·6	1·31	6·50	0·0382	2·175
+13·5	~0	1·27	9·75	~0	1·960
+15·0	~0	1·18	10·80	~0	1·820
+18·0	~0	1·04	13·00	~0	1·600

Table 5

$x = 4$ cm: $\xi = 0{\cdot}551\, y$; $f' = 0{\cdot}0556\, u$; $f = 2{\cdot}02\, v + \frac{2}{3}\xi f'$

y (mm)	u (cm/s)	v (cm/s)	ξ	f'	f'
−17·9	—	0·90	9·77	—	1·818
−15·5	0	—	8·47	0	—
−13·3	0·3	0·91	7·26	0·017	1·920
−10·8	1·15	0·98	5·89	0·064	2·231
−8·8	1·8	0·90	4·81	0·100	2·138
−7·6	2·6	0·86	4·15	0·145	2·138
−7·25	3·3	0·82	3·96	0·183	2·137
−6·9	3·5	0·75	3·77	0·195	2·005
−6·4	4·7	0·65	3·49	0·264	1·926
−5·4	5·1	—	2·95	0·284	—
−4·4	7·8	0·42	2·40	0·434	1·541
−4·0	9·8	0·35	2·18	0·545	1·499
−3·7	10·0	0·30	2·02	0·556	1·353
+0·3	15·8	—	0·164	0·878	0·961
+0·3	15·8	—	0·164	0·878	0·961
+1·3	14·9	—	0·710	0·828	0·484
+1·8	13·7	—	0·983	0·762	—
−2·6	12·8	—	1·42	0·711	—
+3·3	11·4	0·3	1·80	0·634	1·368
+3·6	9·25	0·39	1·97	0·514	1·470
+4·2	8·0	0·39	2·29	0·445	1·483
+5·4	5·3	0·74	2·95	0·294	2·472
+6·8	3·5	0·77	3·71	0·195	2·037
+7·5	—	0·84	4·08	—	—
+8·4	1·6	0·77	4·59	0·090	1·830
+9·8	0·8	0·92	5·35	0·044	2·010
+12·8	0·3	0·93	6·99	0·017	1·996
+16·5	0	—	9·00	0	—
+17·7	—	0·83	9·66	—	1·678

Table 6

$x = 8$ cm: $\xi = 0{\cdot}415\,y; f' = 0{\cdot}0478\,u; f = 2{\cdot}67\,v + \frac{3}{5}\xi f'$

y (mm)	u (cm/s)	v (cm/s)	ξ	f'	f
−16·4	0·9	0·65	6·84	0·043	1·916
−13·4	1·2	0·55	5·59	0·057	1·670
−12·5	1·4	0·55	5·21	0·067	1·687
−11·7	1·8	0·55	4·88	0·086	1·737
−10·7	1·95	0·55	4·46	0·093	1·734
−9·2	2·4	0·43	3·84	0·114	1·431
−8·0	4·75	0·41	3·33	0·226	1·587
−5·6	7·8	0·34	2·34	0·372	1·479
−2·9	11·4	0·25	1·210	0·543	1·099
−0·8	16·7	0·06	0·333	0·795	0·191
−0·55	17·45	—	0·229	0·831	—
0	17·2	—	0	0·819	—
+1·55	15·3	—	0·646	0·729	—
+3·0	13·25	0·1	1·250	0·631	0·788
+5·2	12·0	0·2	2·17	0·571	1·354
+6·75	9·3	0·25	2·81	0·443	1·505
+7·9	6·9	0·24	3·29	0·329	1·356
+9·3	4·8	0·34	3·88	0·229	1·492
+10·9	4·2	0·45	4·54	0·200	1·798
+13·0	2·1	0·49	5·42	0·100	1·660
+14·8	1·4	0·52	6·17	0·067	1·653
+18·1	0·65	0·51	7·55	0·031	1·506

Résumé—Les distributions de vitesse de température déterminées expérimentalement sont présentées pour l'écoulement de convection naturelle dans l'air au-dessus d'un fil horizontal, avec un flux de chaleur constant. La méthode employée dans les mesures de vitesses est une technique de visualisation des lignes de courant au moyen de petites particules de poussière introduites dans l'air.

La distribution de température est déterminée à partir d'un interférogramme.

Zusammenfassung—Für freie Konvektion in Luft an einem waagerechten Draht mit konstantem Wärmefluss werden experimentell ermittelte Geschwindigkeits- und Temperaturverteilungen angegeben. Die für die Geschwindigkeitsmessungen verwendete Methode beruht auf der Sichtbarmachung der Stromlinien mit Hilfe kleiner, in die Luft eingebrachter Staubpartikel.

Die Temperaturverteilung wird aus einem Interferogramm bestimmt.

Аннотация—Представлены результаты экспериментального исследования распределения скоростей и температуры при обтекании воздухом горизонтальной проволоки в условиях свободной конвекции и постоянного теплового потока. Метод, используемый при измерении, состоит в визуализации струй с помощью мелких пылевых частиц, добавляемых к воздуху.

Распределение температуры определяется по интерферограммам.